

# Meson mass and confinement force driven by dilaton

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## Abstract

Meson spectra given as fluctuations of a D7 brane are studied under the background driven by the dilaton. This leads to a dual gauge theory with quark confinement due to the gauge condensate. We find that the effect of the gauge condensate on the meson spectrum is essential in order to make a realistic hadron spectrum in the non-supersymmetric case. In the supersymmetric case, however, only the spectra of the scalars are affected, but they are changed in an opposite way compared to the non-supersymmetric case.

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# 1 Introduction

In the context of the gauge/gravity correspondence of the superstring theory [1], an idea to introduce the flavor quarks has been proposed by Karch and Katz [2]. According to this idea, it becomes possible to study the “meson” spectrum by embedding a D brane in an appropriate 10d background by identifying mesons as brane fluctuations [3, 4, 5, 6, 7, 8].

In particular, the authors in [3] have given a complete analysis for the spectrum of the fluctuations on a D7 brane embedded in  $\text{AdS}_5 \times S^5$ . They represent the mass spectra of mesons formed in a  $\mathcal{N} = 2$  SYM theory. In this gauge theory, however, the potential between quark and anti-quark is Coulomb like at large distance, so the gauge theory is not in the confinement phase.

It would be meaningful to extend this analysis to the case of a background which is dual to a gauge theory with confinement. We consider here a background which is deformed by the dilaton and the axion. In this case, quark confinement is realized in the sense that a linear-rising potential is found at large distances. And the QCD string tension in this model is proportional to the gauge condensate. This point is assured by the running gauge-coupling constant, which diverges in the infrared limit. This force should be responsible for forming the meson states of quark and anti-quark connected by the QCD string. It would be important to see how this point follows from the meson spectra.

Our purpose here is to perform the analysis from the gravity side in the background stated above. In a given background, a D7-brane probe is embedded to add the flavor quark. Then, by using the embedding configuration of the D7 brane, the mass spectra of mesons are examined through the fluctuations on the embedded D7-brane. The analysis is performed first for a supersymmetric background as given in [9, 10]. In this background there is no singularity, and the quark confinement has been assured for heavy quarks [9, 10] and also for flavor quarks. Then we extend the same analysis to a non-supersymmetric case where chiral symmetry is also broken.

In section 2, we give the setting of our model and the embedding of a D7 brane in the supersymmetric background, and we study the mass spectra of the mesons. In section 3 the analysis is extended to the non-supersymmetric case. Summary and discussion are given in the final section.

## 2 Background geometry and embedding of a D7 brane

The D7 brane embedding is briefly reviewed for two types of backgrounds, supersymmetric and non-symmetric, as given in [11].

## 2.1 Supersymmetric background

We consider the  $ISO(1,3) \times SO(6)$  symmetric solution given in [9, 10] for 10d IIB model. This solution is supersymmetric and it has no singularity in the bulk, so we can study the dual gauge theory through the semi-classical approach to bulk gravity. In the present case, the dual gauge theory for this background preserves  $\mathcal{N} = 2$  supersymmetry. The solution can be written in the string frame, taking  $\alpha' = g_s = 1$ , as follows,

$$ds_{10}^2 = G_{MN} dX^M dX^N = e^{\Phi/2} \left( \frac{r^2}{R^2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2 \right), \quad (1)$$

$$e^\Phi = \left( 1 + \frac{q}{r^4} \right), \quad \chi = -e^{-\Phi} + \chi_0, \quad (2)$$

where  $M, N = 0 \sim 9$ ,  $x^\mu = X^\mu (\mu, \nu = 0 \sim 3)$ ,  $R^4 = 4\pi N g_s$  and  $q$  is a constant.  $\Phi$  and  $\chi$  denote the dilaton and the axion respectively, and the self-dual five form  $F_{\mu_1 \dots \mu_5}$  is given as in [9, 10]. And other field configurations are not used here. This solution connects two asymptotic geometries,  $AdS_5 \times S^5$  and flat space-time, respectively, in IR ( $r = 0$ ) limit and UV ( $r = \infty$ ) [9, 10].

From the holographic context, we give a comment on the form of the dilaton  $e^\Phi$ . It represents the running coupling of the dual gauge theory, and the parameter  $q$  is related to the gauge field condensate via  $q/(4\pi N) = \pi^2 \langle F_{\mu\nu} F^{\mu\nu} \rangle$  [10]. So the parameter  $q$  is an important factor which characterizes the vacuum structure of the dual gauge theory, according to the present model.

Next, we introduce the flavor quark by embedding a D7 brane probe which lies in the  $\{x^\mu, X^4 \sim X^7\}$  directions. Here we rewrite the 6d geometry as  $\sum_{M=4}^9 (dX^M)^2 = dr^2 + r^2 d\Omega_5^2 = d\rho^2 + \rho^2 d\Omega_3^2 + (dX^8)^2 + (dX^9)^2$ , where  $\rho^2 = \sum_{M=4}^7 (X^M)^2$ . Then  $r^2 = \rho^2 + (X^8)^2 + (X^9)^2$ . The bosonic part of the brane action for the D7-probe is

$$S_{D7} = -\tau_7 \int d^8 \xi \left( e^{-\Phi} \sqrt{-\det(\mathcal{G}_{ab} + 2\pi\alpha' F_{ab})} + \frac{1}{8!} \epsilon^{i_1 \dots i_8} A_{i_1 \dots i_8} \right) + \frac{(2\pi\alpha')^2}{2} \tau_7 \int P[C^{(4)}] \wedge F \wedge F, \quad (3)$$

where  $F_{ab} = \partial_a A_b - \partial_b A_a$ .  $\mathcal{G}_{ab} = \partial_{\xi^a} X^M \partial_{\xi^b} X^N G_{MN}$  ( $a, b = 0 \sim 7$ ) and  $\tau_7 = [(2\pi)^7 g_s \alpha'^4]^{-1}$  represent the induced metric and the tension of D7 brane respectively. And  $P[C^{(4)}]$  denotes the pullback of a bulk four form potential,

$$C^{(4)} = \left( \frac{r^4}{R^4} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \right). \quad (4)$$

The eight form potential  $A_{i_1 \dots i_8}$ , which is the Hodge dual to the axion, couples to the D7 brane minimally. In terms of the Hodge dual field strength,  $F_{(9)} = dA_{(8)}$  [12], the potential  $A_{(8)}$  is obtained as  $A_{i_1 \dots i_8} = -e^\Phi \epsilon_{i_1 \dots i_8}$ .

By taking the canonical gauge,  $\xi^a = X^a$ , we find an embedding by solving the equation of motion for the fields  $X^8(\xi)$  and  $X^9(\xi)$  under the ansatz,  $X^9 \equiv w(\rho)$  and

$X^8 = 0 = F_{ab}$  [11]. The stable solution is the supersymmetric one of constant  $w$ , which could take an arbitrary value [11]. And the induced metric is written as

$$ds_8^2 = e^{\Phi/2} g_{ab} dx^a dx^b = e^{\Phi/2} \left( \frac{r^2}{R^2} dx^\alpha dx_\alpha + \frac{1}{r^2} R^2 d\rho^2 + \frac{\rho^2}{r^2} R^2 d\Omega_3^2 \right) \quad (5)$$

where  $r^2 = \rho^2 + w^2$  and  $g_{ab}$  represents the Einstein frame metric.

Then, the fluctuations of the fields on D7 brane are set as follows,

$$X^9 = w + \phi^9, \quad X^8 = \phi^8 \quad \text{and} \quad A_a, \quad (6)$$

and the D7 action is expanded in them up to the quadratic part,

$$\begin{aligned} \mathcal{L} = -\tau_7 \left\{ e^\Phi \sqrt{-\det g_{ab}} \left( \frac{1}{2} g^{ab} \frac{R^2}{r^2} \sum_{i=8}^9 (\partial_a \phi^i \partial_b \phi^i) + e^{-\Phi} \frac{(2\pi\alpha')^2}{4} g^{ab} g^{cd} F_{ac} F_{bd} \right) \right. \\ \left. - \frac{(2\pi\alpha')^2}{8} \frac{r^4}{R^4} \epsilon^{\mu\nu\lambda\sigma} F_{\mu\nu} F_{\lambda\sigma} + \dots \right\}. \quad (7) \end{aligned}$$

In the case of  $\Phi = 0$ , the addition of a D7-brane breaks the supersymmetry to  $\mathcal{N} = 2$ . The light modes coming from strings connecting the D3-branes and the D7-brane give rise to a  $\mathcal{N} = 2$  hypermultiplet in the fundamental representation, whose field content is two complex scalar fields  $\phi^m$  and two Weyl fermions  $\psi_\pm$ , of opposite chirality.

Here we consider a non-trivial dilaton, so the supersymmetry is further broken to  $\mathcal{N} = 1$  due to the gauge condensate  $q$ . In the following, we study the meson spectrum to see the effect of the gauge condensate for the supersymmetric case.

## 2.2 Scalar field fluctuation

First of all, we consider the scalars  $\phi_i$ . They are expressed by the same linearized equation of motion,

$$\partial_a \left( e^\Phi \sqrt{-\det g_{ab}} g^{ab} \frac{R^2}{r^2} \partial_b \delta X \right) = 0 \quad (8)$$

where we abbreviate  $\phi_8 = \phi_9 = \delta X$ . We write  $\delta X$  as

$$\delta X = \phi(\rho) e^{ik \cdot x} \mathcal{Y}^l(S^3), \quad (9)$$

where  $\mathcal{Y}^l(S^3)$  are the scalar spherical harmonics on  $S^3$ , which transform in the  $(l/2, l/2)$  representation of  $SO(4)$  and satisfy

$$\nabla^i \nabla_i \mathcal{Y}^l = -l(l+2) \mathcal{Y}^l. \quad (10)$$

Then equation (8) becomes

$$\partial_\rho^2 \phi + \frac{3}{\rho} \partial_\rho \phi + \left( \frac{(mR^2)^2}{(w^2 + \rho^2)^2} - \frac{l(l+2)}{\rho^2} \right) \phi + \partial_\rho \Phi \partial_\rho \phi = 0, \quad (11)$$

where  $-k^2 = m^2$ , which denotes the four dimensional mass. This equation reduces to the one given in [3] for  $\Phi = 0$ . The part depending on the dilaton is written as

$$\partial_\rho \Phi = -\frac{4q\rho}{(w^2 + \rho^2)(q + (w^2 + \rho^2)^2)}, \quad (12)$$

and it is negligible near  $\rho \rightarrow 0$  compared to  $3/\rho$ . So the infrared behavior of this equation is not changed by this dilaton term. As for the UV limit, this term is again negligible, and we obtain the following asymptotic solution,

$$\phi \sim A\rho^{-l-2} + B\rho^l. \quad (13)$$

From this, we can see that the conformal dimension of this mode is  $\Delta = 3 + l$  as expected.

So the effect of the dilaton or of  $q = \langle F_{\mu\nu}^2 \rangle$  would be seen in the eigenvalue of the meson mass. For  $q = 0$ , the analytic solution has been given in [3] and it will be a good approximation at  $\rho \rightarrow 0$  and  $\rho \rightarrow \infty$  for  $q > 0$  as mentioned above. Then we can solve the equation numerically by using the analytic solution for  $q = 0$  as the boundary conditions. The numerical results are shown in Fig.1 for the mass in the case of  $l = 0$ .

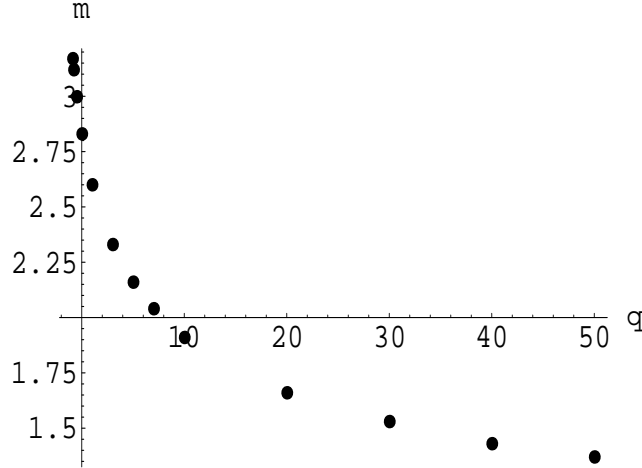


Fig. 1: Meson mass  $m(q)$  vs gauge condensate  $q$ , where  $m_q = 1/(2\pi\alpha')$  and  $R = 1$ .

The value of the mass decreases with increasing  $q$ . This is understandable from the viewpoint of the string theory. In the present case, the potential between quark and anti-quark is linear with the distance between quarks at large distances. And its tension is proportional to  $\sqrt{q}$ . Then we expect that  $m^2 \propto 1/\sqrt{q}$ . As we are now considering the small mass meson states, this relation would be only approximate. However, we can see the qualitative feature of this relation from our results.

### 2.3 Gauge field fluctuations

The equations of motion for the gauge fields on the D7-brane, which follow from the action (7), are independent of  $\Phi$ . They are given as

$$\partial_a(\sqrt{-\det g_{cd}} F^{ab}) - \frac{4\rho(\rho^2 + w^2)}{R^4} \varepsilon^{bjk} \partial_j A_k = 0, \quad (14)$$

where  $\varepsilon^{ijk}$  is a tensor density (i.e., it takes values  $\pm 1$ ). The second term comes from the Wess-Zumino part of the action, proportional to the pullback of the RR five-form field strength, and is present only if  $b$  is one of the  $S^3$  indices.

Then the gauge field fluctuations are not affected by the dilaton. This seems to be very strange since the constituent quarks of the meson state would be influenced by the gauge interaction characterized in terms of the dilaton. But the above equation implies that the force responsible to make the meson bound state is not changed from the one which could not confine quarks. And the confining force, which is responsible to produce the linear potential between quark and anti-quark, affects only the scalar fluctuations.

In this sense, there are many kind of bound states in the present model. The quark and anti-quark in the vector mesons can be separated at long distances from each other since they are bounded by the Coulomb-like potential at large distance. But a quark can not be singled out as a free quark state because it would have an infinite mass. This can be understood from the Wilson loop, which is taken in the  $t - r$  plane, as follows. The relevant part of the metric for this Wilson loop is

$$ds^2 = e^{\Phi/2} \left( -\frac{r^2}{R^2} dt^2 + \frac{R^2}{r^2} dr^2 \right). \quad (15)$$

Then the action for the  $t - r$  Wilson loop is obtained as

$$S_{t-r} = \frac{1}{2\pi} \int dt dr e^{\Phi/2}, \quad (16)$$

and the value of this action for a unit time represents the effective quark mass,  $\tilde{m}_q$ . It is obtained as

$$\tilde{m}_q = \frac{1}{2\pi} \int dr e^{\Phi/2} = \frac{1}{2\pi} \int_0^{r_{max}} dr \sqrt{1 + \frac{q}{r^4}}. \quad (17)$$

This implies  $\tilde{m}_q = \infty$  for any finite  $m_q = r_{max}/2\pi$ , which corresponds to the current quark mass. Then it would be impossible to observe a single quark. In other words, the quark is confined.

### 3 Non-supersymmetric background and meson mass

Here we consider a non-supersymmetric solution [13, 21, 15] which is given without changing the five form field and eliminating the axion,  $\chi = 0$ , as,

$$ds_{10}^2 = e^{\Phi/2} \left( \frac{r^2}{R^2} A^2(r) \eta_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2 \right), \quad (18)$$

$$A(r) = \left( 1 - \left( \frac{r_0}{r} \right)^8 \right)^{1/4}, \quad e^\Phi = \left( \frac{(r/r_0)^4 + 1}{(r/r_0)^4 - 1} \right)^{\sqrt{3}/2}. \quad (19)$$

For  $\chi = 0$ , two dilaton solutions  $\Phi$  are possible and they correspond to the magnetic and electric gauge field condensate respectively. Actually,  $e^\Phi$  are expanded as

$$e^\Phi \sim 1 + \frac{q_{\text{NS}}}{r^4}, \quad q_{\text{NS}} = \sqrt{6} r_0^4. \quad (20)$$

As shown in the previous case, in the context of AdS/CFT, we can interpret the parameter  $q_{\text{NS}}$  as the gauge-field condensate  $\langle F_{\mu\nu} F^{\mu\nu} \rangle = \langle H_i^2 - E_i^2 \rangle$ .

These configuration have a singularity at the horizon  $r = r_0$ , and the present semi-classical analysis can not be applied near this point. So we avoid this point in the following.

The D7 brane is embedded as above, and the following brane action is obtained,

$$\begin{aligned} S_{\text{D7-NS}} &= -\tau_7 \int d^8 \xi \mathcal{L}_{\text{NS}} \\ &= -\tau_7 \int d^8 \xi \sqrt{\epsilon_3} \rho^3 e^\Phi \left( 1 - \left( \frac{r_0}{r} \right)^8 \right) \sqrt{1 + (w')^2}. \end{aligned} \quad (21)$$

We could expect to find SCSB by solving the equation of motion for  $w$ ,

$$w'' + (1 + w'^2) \left\{ \frac{3}{\rho} w' + 2K_{(1)} \left[ w'(\rho + ww') - (1 + w'^2)w \right] \right\} = 0, \quad (22)$$

$$K_{(1)} = \frac{1}{e^\Phi A^4} \partial_{r^2} (e^\Phi A^4). \quad (23)$$

For some appropriate solution of this equation, we examine the meson mass by changing the gauge field condensate  $q_{\text{NS}}$  to see the effects of the gauge condensate on the meson mass as given in the previous section for the supersymmetric case.

#### 3.1 Scalar meson

With

$$X^8 = \phi^8, \quad X^9 = w(\rho) + \phi^9$$

the field equations for  $\phi^8$  and  $\phi^9$  are

$$\partial_\rho^2 \phi^8 + \frac{1}{L_0} \partial_\rho(L_0) \partial_\rho \phi^8 + (1 + w'^2) \left[ \left(\frac{R}{r}\right)^4 \frac{m^2}{A^2} - \frac{l(l+2)}{\rho^2} - 2K_{(1)} \right] \phi^8 = 0 \quad (24)$$

$$L_0 = \rho^3 e^\Phi A^4 \frac{1}{\sqrt{1 + w'^2}}, \quad (25)$$

and

$$\begin{aligned} \partial_\rho^2 \phi^9 + \frac{1}{L_1} \partial_\rho(L_1) \partial_\rho \phi^9 + (1 + w'^2) \left[ \left(\frac{R}{r}\right)^4 \frac{m^2}{A^2} - \frac{l(l+2)}{\rho^2} - 2(1 + w'^2)(K_{(1)} + 2w^2 K_{(2)}) \right] \phi^9 \\ = -2 \frac{1}{L_1} \partial_\rho(L_1 w' w K_{(1)}) \phi^9 \end{aligned} \quad (26)$$

$$L_1 = \frac{L_0}{1 + w'^2}, \quad K_{(2)} = \frac{1}{e^\Phi A^4} \partial_{r^2}^2 (e^\Phi A^4). \quad (27)$$

In deriving these, we used

$$r^2 = \rho^2 + (\phi^8)^2 + (\phi^9)^2 + w^2 + 2w\phi^9 \quad (28)$$

but in the above field equations it is understood as  $r^2 = \rho^2 + w^2$  since we are considering the linearized equations.

In order to see the Nambu-Goldstone (NG) mode, which should be appearing due to the chiral symmetry breaking of the flavor quark, we rewrite the above equation in terms of polar coordinates in  $X^8 - X^9$  plane,

$$X^8 = p \sin(\theta), \quad X^9 = p \cos(\theta), \quad p = w(\rho) + \delta p. \quad (29)$$

Here  $\theta$  and  $\delta p$  are the fluctuations. Then the above equations are rewritten in the linearized approximation as  $X^8 = w\theta$ ,  $X^9 = w + \delta p$ . Then the fluctuations given above are identified as

$$\phi^8 = w\theta, \quad \phi^9 = \delta p. \quad (30)$$

Thus the equation for  $\theta$ , which should be the NG mode, is obtained by substituting  $\phi^8 = w\theta$  into Eq.(24). Then we get

$$\partial_\rho^2 \theta + \left( 2 \frac{w'}{w} + \frac{1}{L_0} \partial_\rho L_0 \right) \partial_\rho \theta + (1 + w'^2) \left[ \frac{R^4 m^2}{r^4 A^2} - \frac{l(l+2)}{\rho^2} \right] \theta = 0 \quad (31)$$

In order to see the lowest mode of  $\theta$ , the s-wave of this mode is studied. Equation (31) is analyzed in the region of large  $\rho$  by using the asymptotic form of  $w(\rho) = m_q + c/\rho^2$ . For large  $\rho$ , then, Eq.(31) is rewritten as

$$\partial_\rho^2 \theta + \left( -4 \frac{c}{m_q \rho^3} + \frac{3}{\rho} \right) \partial_\rho \theta + \frac{R^4 m^2}{\rho^4} \theta = 0, \quad (32)$$



First of all, we consider the case of positive and finite  $m_q$ , where we may find the asymptotic form of the renormalizable solution as

$$\theta = \frac{1}{\rho^2} + \frac{a_1}{\rho^4} + \cdots, \quad a_1 = -\left(\frac{c}{m_q} + \frac{R^4 m^2}{8}\right) \quad (33)$$

From the second equation of (33),

$$m^2 = m_0^2 - \frac{8}{R^4} \frac{c}{m_q}, \quad m_0^2 = -\frac{8a_1}{R^4}. \quad (34)$$

Since  $c/m_q > 0$ ,  $m_0^2$  should be positive and  $m_0^2 > \frac{8}{R^4} \frac{c}{m_q}$  due to the positivity of  $m^2$ . We thus could get a normalizable mode of positive  $m^2$  at some value of  $m_q$ , then we could consider a smaller point of  $m_q$  where  $m^2 \rightarrow 0$  since  $c$  is finite for  $m_q \rightarrow 0$ . So we can say that  $m^2$  decreases with decreasing  $m_q$ , but we can't use the above formula near  $m^2 \sim 0$  since the approximate equation (32) is not correct for small  $m_q$ . However we expect that the minimum of  $m^2$  would be realized at  $m_q = 0$  as shown below.

From Eq.(31), we find a solution of  $m = 0$  for  $l = 0$  as  $\theta = \theta_0$ , which is a constant. Next, we examine the normalizable condition for this solution, it is given as

$$\int_0^\infty d\rho \frac{1}{\rho} (\phi^8)^2 \sim \int_0^\infty d\rho \frac{1}{\rho} (m_q + \frac{c}{\rho^2})^2 (\theta_0)^2 < \infty. \quad (35)$$

This condition is satisfied only for  $m_q = 0$ . In other words, there appears a massless mode only for the case of  $m_q = 0$  and  $c = -\langle \Psi \bar{\Psi} \rangle \neq 0$ . In this sense, this zero mode  $\theta$  corresponds to the pion and the lowest massive-mode of  $\phi^9$  would correspond to the sigma meson, which is massive. We can show these points directly by the numerical estimation given in Fig. 2. In this figure, the lowest mode of  $\phi^8$  and  $\phi^9$  are shown. As expected from the above speculation, the mass of  $\phi^8$  approaches zero with decreasing  $m_q$ , while the one of  $\phi^9$  arrives at a finite value. On the other hand, both masses coincide at large  $m_q$  and rise linearly. At large  $m_q$ ,  $w$  is almost constant and  $w'$  can be neglected, then both equations for  $\phi^8$  and  $\phi^9$  approach the same form when  $K_{(2)}$  is neglected compared to  $K_{(1)}$ . This point is assured directly by the numerical estimation. This is the reason why the masses of  $\phi^8$  and  $\phi^9$  coincide at large  $m_q$ .

The massive modes of  $\theta$  and  $\phi^9$  are obtained by the numerical analysis. In both cases, we find that the value of the masses increases with the strength of the gauge condensate  $q_{\text{NS}}$  given in (20). An example for the case of  $\phi^8$  is shown by the solid curve in Fig.2 for  $r_0 = 2$ , which is larger than the case of  $r_0 = 1$ . This behavior is essentially different from the case of the supersymmetric solution. In the latter case, the mass is suppressed when the gauge condensate  $q$  increases. It would be an interesting problem to understand this difference from the gauge theory side, but the problem is left open here.

## 4 Summary and discussion

The background given by (1) and (2) has been extended to the case of finite temperatures, also with the D7 brane embedded in this background [22]. The fluctuation of

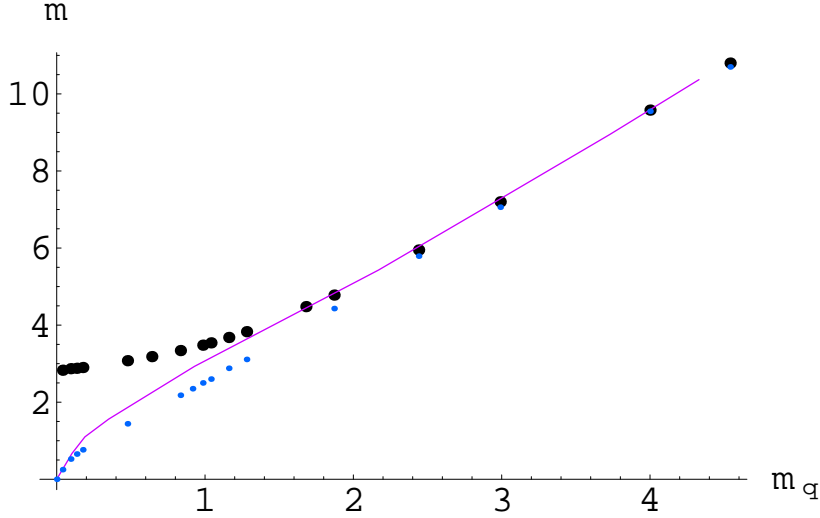


Fig. 2: Meson mass  $m$  for  $\phi^8$  (small circle) and  $\phi^9$  (large circle) vs  $m_q$  for  $r_0 = 1$ . The solid curve shows  $m$  of  $\phi^8$  for  $r_0 = 2$ . Here we take  $R = 1$ .

the D7 brane in this background would give the meson spectra at quark-gluon plasma phase. This would lead to information about the matter properties in the hot universe.

In the context of gauge/gravity correspondence, the meson spectra were studied by embedding a D7 brane in the background deformed by the dilaton. We examined both the supersymmetric and non-supersymmetric background configurations. For the supersymmetric case, the gauge condensate  $q$  affects only the spectrum of the scalar field and the one of the gauge bosons are independent of  $q$ . The mass of the scalar fields decreases with increasing  $q$ . This implies that the mass is suppressed when the gauge coupling constant becomes strong. Then we can suppose that the mechanism to make the bound state in this case is similar to the case of QED.

On the contrary, in the non-supersymmetric case, the gauge condensate affects both on the scalar and vector fields. The spectra of the scalars are examined here and we find that the lowest mass of one of the scalar fields is zero and other modes are massive when quark mass is zero. This is understood as the spontaneous chiral symmetry breaking from the gauge theory side. The masses of the massive modes are increasing with the gauge condensate  $q_{\text{NS}}$ . Since the string tension is proportional to  $q_{\text{NS}}$ , the meson, the bound state of quark and anti-quark, in this case would be connected by the QCD string.

It would be an interesting issue to extend our analysis into the case of finite temperature. The confinement force driven by the dilaton is expected to be remaining even at finite temperature [21, 22], so we will find similar spectra given here for the mesons up to an appropriate temperature before realizing the complete QGP phase. This will leads to finding a new phase transition of QCD at high temperature.

In closing, let us make some speculative remarks on the somewhat related topic of Randall-Sundrum brane scenario [17], assuming that there is only one single brane

present. As is known, all physical processes except for gravity have to be restricted to lie on the brane. Emission of gravitons into the bulk can be imagined to be related to the interaction between particles on the brane,  $\psi + \bar{\psi} \rightarrow G$ . This implies an energy loss equation for ordinary matter, in the form of a Boltzmann equation, where  $C[f]$ , the collision term with  $f$  the distribution function, can be written in the form given explicitly in Refs. [18].

Imagine now that the bulk is not empty but filled with a gas of gravitons. These gravitons may be absorbed or reflected by the brane and exert radiation forces on it. Such a process would be quite analogous to the electromagnetic interaction when a beam of radiation is incident on a fluid surface, as exemplified in the classic experiment of Ashkin and Dziedzic [19]. Actually in electrodynamics, if one uses a phase-separated liquid mixture close to the critical point, whereby the surface tension becomes very small, one can manage to create "giant" displacements, of the order of tenths of micrometers, even with moderate laser powers of a few hundred of milliwatts [20]. The natural question becomes: Is a brane in a higher-dimensional space analogous to a surface in electrodynamics, in the sense that it is flexible and thus subject to deflections caused by incident graviton radiation?

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